Travel Cost Method Considering Trip-day Counts as Integers

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Abstract

Travelers do not choose the number of days spent in a tourist city as continuous numbers but integer numbers such as one night or two nights. The present paper investigates how a bias could arise from ignoring the fact that people stay for integer numbers of nights in the travel cost method. Our theoretical investigation derives the formula of the bias and finds that when the travel time increases, the bias can be either larger or smaller. Next, our numerical simulation demonstrates that when measuring benefits resulting from improving quality at recreation sites, the maximum bias of our numerical simulation could be around 20%.

JEL classification: Q26, Q51

Keywords: Recreation, Travel cost method, Integer property, Trip length, Lodging.

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1. Introduction

The travel cost method (TCM) has been commonly used to estimate an economic value associated with recreation sites, amenities, and benefits resulting from changes in environmental quality at a recreation site. The method employs consumer surplus of people who visit a recreation site. When people take a vacation in a tourist city, such as Tokyo, Paris, or New York City, to visit various recreation sites, they require lodging for several nights. Then, travelers choose the number of days spent in the city (called ‘trip length’, hereafter) as an integer such as one night or two nights. Previous TCM studies, however, ignoring the fact that people decide their activities in units of a day, treat this as a continuous variable, which may result in biased benefit estimates.

The present paper investigates how the bias could arise from ignoring the integer property of trip length. We develop a travel cost model which treats the endogenous trip length as an integer variable. Our theoretical analysis derives the formula representing what constitutes the bias and finds that it can be either larger or smaller as the travel time to a destination city increases. In addition to the theoretical analyses, we numerically simulate the bias by using the model with parameters calibrated with a data set of visits to Hokkaido in Japan. When measuring benefits resulting from improving quality at recreation sites such as improving facilities and adding entertainment attractions to amusement parks, a numerical simulation results in up to a 14% underestimate and a 11% overestimate in the case of the Hokkaido case, depending on the travel time. In addition, our sensitivity analyses show that the maximum bias could be around 20%.

Trip length was first discussed in TCM to find whether lodging expenses should be included in the travel costs (Randall, 1994). Lodging and related expenses were considered in the model of Kealy and Bishop (1986). They assume travelers choose the total number of visits, but the trip length is exogenous and constant across travelers. English and Bowker (1996) explored the sensitivity of consumer surplus with and without lodging expenses. On the other hand, regarding whether trip length should be treated as endogenous or exogenous, McConnell (1992) showed that the effect of treating this as an endogenous variable on welfare measurements is negligible.

In actual trips, travelers can choose trip length to enhance enjoyment of recreational activities. In a single site model, Larson (1993) developed a model where a traveler simultaneously chooses the numbers of trips and trip length. Landry and McConnell (2007) extended the previous models to the situation where a traveler chooses quality derived through accommodations, dining services, and recreational activities. They suggested that such quality choices affect TCM estimates, and the overall effect on welfare estimation is significant. In a multiple-site model, Berman and Kim (1999) and Yeh et al. (2006) treat the trip length at a recreation site as an endogenous variable.

Previous studies, however, do not sufficiently discuss how to treat trip length in TCM[1]. They posit that a traveler chooses the trip length or the total on-site time as a continuous variable. But during the actual trip, because humans determine their activities in units of a day, he or she chooses it as an integer number such as one night or two nights[2]. For example, according to the Kyoto official report on tourism (2013), 52 % of visitors who stay overnight in Kyoto stay for one night, 34 % stay two nights, and almost 10 % stay three nights. Depending on how many nights they stay, their available time for leisure changes discretely. Accordingly, in order to measure benefits, travel cost method should take account of these discrete changes in the available time.

The main purpose of this study is to understand how and how much a bias in benefit

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[1] Takeuchi (1999) also pointed out several difficulties in applying TCM to actual situations. But integer number constraints for trips, which the current paper targets, is not pointed out.

[2] Discrete decision of when and where to make trips with the continuous decision of how much to make some activities subject to a budget constraint can be taken into account by Kuhn–Tucker demand models (e.g., Phaneuf 1999, and Kuriyama et al. 2010). But these models also do not take account of the integer property of trip length.

estimates could arise from ignoring the integer property of trip length and how the bias changes when the travel time increases. Focusing on this problem, we ignore other problems with TCM (e.g., problems raised by Randall (1994)). We develop a model where a traveler chooses the trip length as an integer variable to visit several recreation sites, but the length of time spent at each recreation site is treated as a continuous variable.

The integer property of endogenous variables in TCM has been surveyed by Dobbs (1993). He points out that the bias may arise from ignoring the integer property of the number of visits. Namely, when the number of visits to the site is regarded as discrete events, the benefit estimated by the integration of smooth demand function could have a bias. Even though the ordinary least square has often been used for estimating the demand function, it fails to account for the integer property of variables or count data, which results in an upward biased estimate (e.g. Shrestha et al., 2002; Nahman and Rigby, 2008).

As well as the number of visits, trip length should be treated as a discrete variable, such as a two-day or three-day trip. Our theoretical analysis shows that when the travel time increases, the bias can be either larger or smaller. An intuitive reason for the bias arising is as follows. When travel time increases, the trip length that a traveler actually chooses is fixed unless he or she decides to stay one more night. Since the trip length is fixed but the travel time increases, a traveler cannot avoid decreasing time spent on visiting recreation sites. Contrary to an integer trip length (measured in days), a continuous trip length can smoothly change according to changes in travel time. This allows time spent on visiting recreation sites to smoothly change even if the travel time changes. Such differences between integer and continuous trip length cause the bias.

To explore such integer biases theoretically, we explore the utility maximization model with explicit consideration of time constraints as DeSerpa (1971) and Larson and Lew (2014). The household taking a trip to a tourist city to visit some recreation sites chooses the time spent in each recreation site and the number of visiting sites. Under the integer trip length model, the first order condition for the trip length cannot be used. The household chooses the integer trip length such that the indirect utility is maximized.

Some recent TCM approaches employ random utility models (e.g., Berman and Kim (1999) and Yeh et al. (2006)). In these approaches, indirect utility functions are specified and then the parameters are estimated using trip data. They can consider trip length as an endogenous variable for such approaches. However, in most cases, the specified indirect utility function is not based on a time constraint. So, the indirect utility function does not reflect the integer property of trip length appropriately. To focus on the mechanisms of generating biases, we show how the original TCM approach proposed by Hotelling (1947), which uses demand functions, causes biases due to the integer property of trip length.

The rest of the paper is as follows. Section 2 constructs a model treating trip length as an integer variable. Section 3 theoretically analyzes how the bias in benefit estimates could arise from using a continuous trip length when the travel time increases. Section 4 quantifies the bias in benefit estimates resulting from an improvement in quality at recreation sites through a numerical simulation. Section 5 concludes the paper.

2. Travel Cost Model with Integer Trip length

A household spends the total time endowment $\overline{T}$, excluding sleeping time, on working T_w, taking a trip to a tourist city to visit some recreation sites, and other leisure activities *l*. Hence, the full time constraint is

$$\overline{T} = T_w + \delta n \overline{T}_t + l, \tag{1}$$

where $\overline{T}_t$ is exogenously daily available time excluding the time for sleeping and breakfast on a trip (e.g. 24 hours – 14 hours for sleeping and breakfast = 10 hours spent on sightseeing), *n* is the number of days spent in the city, and δ is a dummy variable of whether the household travels.

A household which lives far from the destination city must stay a few days to visit several recreation sites. In such a situation, n changes by an integer number of days. Here, we formally define n as follows.

Definition 1. A household chooses n as a non-negative *integer* variable.

On an n-day trip, the household spends the available time $n\overline{T}_t$ on traveling between the origin and the destination city, visiting k number of recreation sites, and other activities l_1, such as leisure time after returning home on the last day of the trip. Therefore, the household's time constraint for the trip is

$$\delta n\overline{T}_t = \delta\left(t_0 + \sum_{i=1}^{k} t_i + l_1\right), \tag{2}$$

where t_0 is a round-trip time between the origin and the destination city, t_i is time spent on visiting recreation site i, and i is the name of each recreation site. Since n is an integer variable, (2) indicates that the available time per day on a trip discretely increases with n. In other words, people sleep every day, so the available time for visiting leisure sites changes discretely with number of days.

The household earns wage w per hour and consumes numeraire composite goods X. The household pays the cost of traveling between the origin and the destination city, visiting k number of recreation sites, and staying at the city for $n-1$ nights. Thus, the income constraint is

$$wT_w = X + \delta\left[p_0 t_0 + \sum_{i=1}^{k} p_i + (n-1)p_h\right], \tag{3}$$

where p_0 is the unit traveling expense between the origin and the destination, p_i is the cost of visiting recreation site i, and p_h is daily overnight expenditure.

The household obtains utility from visiting k kinds of recreation sites, staying for t_i hours at each site, consuming X, and leisure activities other than trips l. In reality, t_i differs among sites, but our analysis assumes that t_i is common just for simple exposition.[3] Even if we assume a different t_i, we essentially obtain the same proposition. Furthermore, utility is assumed to be additively separable into these variables for simplicity. Therefore, the household's utility function is given by

$$V = \delta\big(U(t_i,\ k;\ q) + l_1\big) + S(X) + l, \tag{4}$$

where $U(t_i,\ k;\ q)$ is utility from visiting recreation sites, q is the quality of the tourist city which is determined by the quality of all recreation sites in the city, and $S(X)$ is utility from consuming X. We assume $U_{t_i} \equiv \partial U/\partial t_i > 0$, $U_k \equiv \partial U/\partial k > 0$, and $\partial^2 U/\partial t_i \partial k = 0$.[4]

The household maximizes the utility function (4) subject to (1)–(3) and $l_1 \geq 0$:

$$\max_{\delta, t_i, k, T_h, l, T_w, n, X} \delta\big(U(t_i,\ k;\ q) + l_1\big) + S(X) + l \ \text{ s.t. (1), (2), (3) and } l_1 \geq 0. \tag{5}$$

If the leisure is more valuable than the opportunity cost for time, travelers spend the last day of their trip on enjoying the trip only, and they do not go home early for other activities in their origin site. This study, for simplicity, assumes that l_1 equals zero hereafter.

Substituting (1) and (3) into (4) to simplify (5), we can obtain

$$\max_{\delta, t_i, k, T_h, n, X} w\delta U(t_i,\ k;\ q) + wS(X) + w\overline{T} - X - \delta\left(p_0 t_0 + \sum_{i=1}^{k} p_i + (n-1)p_h\right) - \delta w n \overline{T}_t \tag{6}$$

s.t. (2).

We solve the utility maximization problem in two steps: (i) optimizing all variables except n, (ii) obtaining optimal n so that indirect utility is maximized. We define the Lagrangian function

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[3] In addition to the current model, we analyze a case where the household derives utility from accommodation. In that case, the household chooses time spent at a lodging place. The results are available from the authors.

[4] $\partial^2 U_S/\partial t_i \partial k > 0$ if there is a relationship among recreation sites such as two different types of castles created by a historical figure (so, a traveler can enjoy comparing two castles). $\partial^2 U_S/\partial t_i \partial k < 0$ if there are many similar recreation sites. However, we assume that $\partial^2 U_S/\partial t_i \partial k = 0$ for simplicity. Actually, this can be justified to some extent because time spent at a recreation site is different from time spent at another site so that the relationship between the two sites is not so strong.

as

$$
\begin{aligned}
L &= w\delta U(t_i,\ k;\ q) + wS(X) + w\overline{T} - X \\
&\quad -\delta\left(p_0 t_0 + \sum_{i=1}^{k} p_i + (n-1)p_h \right) - \delta wn\overline{T}_t - \lambda w\delta\left(t_0 + \sum_{i=1}^{k} t_i - n\overline{T}_t \right),
\end{aligned}
\tag{7}
$$

where λ is a Lagrangian multiplier of (2). First, differentiating (7) with respect to all endogenous variables except n, we obtain the following first order conditions:

$$\frac{\partial L}{\partial t_i} = w\delta\left(U_{t_i} - \lambda\right) = 0, \tag{8}$$

$$\frac{\partial L}{\partial k} = w\delta\left(U_k - \frac{p_i}{w} - \lambda \cdot t_i \right) = 0, \tag{9}$$

$$\frac{\partial L}{\partial X} = w\frac{\partial S(X)}{\partial X} - 1 = 0, \text{ and} \tag{10}$$

$$\frac{\partial L}{\partial \lambda} = -w\delta\left(t_0 + \sum_{i=1}^{k} t_i - n\overline{T}_t \right) = 0. \tag{11}$$

Second, we optimize n. Substituting all variables optimized in the first step into the utility function, we obtain the following indirect utility function:[5]

$$
\begin{aligned}
\max_{\delta,n} V(n;\ q) &= w\delta U\left(t_i^*(n;\ q),\ k^*(n;\ q)\right) + wS(X^*) + w\overline{T} - X^* \\
&\quad -\delta\left(p_0 t_0 + \sum_{i=1}^{k^*(n;\ q)} p_i + (n-1)p_h \right) - w\delta n\overline{T}_t.
\end{aligned}
\tag{12}
$$

Since we regard n as an integer variable, the first order condition with respect to n cannot be used. We define the optimal integer trip length n^* as follows.

Definition 2. $\quad n^* = \arg\max\left[V(n;\ q) : n \in N\right]$ (13)

Next, we derive the recreational benefit of visiting a tourist city with an integer trip length

[5] Hereafter, "*" after variables represents the optimal variable.

denoted as B_d. It is defined as the difference between the indirect utility with and without the trip to the city,

$$B_d(q) \equiv \Delta V(n^*) = V\left(n^* : \delta = 1\right) - V\left(n^* : \delta = 0\right),$$

where

$$V\left(n^* : \delta = 1\right) = wU\left(t_i^*(n^*), k^*(n^*);\ q\right) + wS(X^*) + w\overline{T} - X^* - p_0 t_0$$
$$- \sum_{i=1}^{k^*(n^*)} p_i - (n^* - 1)p_h - wn^*\overline{T}_t, \text{ and}$$
$$V\left(n^* : \delta = 0\right) = wS(X^*) + w\overline{T} - X^*.$$

Therefore, B_d is given by[6]

$$B_d(q) = wU\left(t_i^*(n^*), k^*(n^*);\ q\right) - p_0 t_0 - \sum_{i=1}^{k^*(n^*)} p_i - (n^* - 1)p_h - wn^*\overline{T}_t. \qquad (14)$$

The next section is devoted to showing the benefit estimates with continuous trip lengths and their biases.

3. Bias due to Ignoring the Integer Property of Trip Length

This section analyzes how benefit estimates cause biases due to ignoring the integer property of trip length. To answer this question, we treat n as a continuous variable in the model in section 2. Then we compare its result to the result with an integer trip length. Hereafter, to easily recognize the continuous or the integer trip length, we denote them as n_c and n_d, respectively.

3.1 Model with continuous trip length

With a continuous trip length, the left hand side of (2) continuously changes, and we can use the first order condition with respect to n_c. Only this point is different from the model in section 2. Differentiating the same Lagrangian function (7) with respect to t_i, k, X, n_c, and λ, we can

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[6] The subscript "*d*" in B stands for "discrete" and indicates the benefits with an integer trip length.

obtain first order conditions. The first order conditions with respect to t_i, k, X, and λ are the same as in section 2, i.e. (8)–(11) hold. The different first order condition—the first order condition with respect to n_c —is given by

$$\frac{\partial L}{\partial n_c} = w\delta\left(-\frac{p_h}{w} - \overline{T}_t + \lambda\overline{T}_t\right) = 0. \tag{15}$$

From the first order conditions with respect to t_i, k, X, n_c, and λ, we can obtain all optimal variables. The recreational benefit with a continuous trip length B_c is defined as the difference between the indirect utility with and without the trip. B_c is given by[7]

$$B_c(q) \equiv wU(t_i^*, k^*; q) - p_0 t_0 - \sum_{i=1}^{k^*} p_i - (n_c^* - 1)p_h - wn_c^*\overline{T}_t. \tag{16}$$

B_c is larger than B_d because all variables are continuous in B_c; that is, B_c is always an overestimation. The question is how the bias changes as the round-trip time increases. We define the difference between them as

$$\Delta \equiv B_c(q) - B_d(q) \geq 0\,. \tag{17}$$

When the travel time increases, the trip length that the traveler actually chooses is fixed unless the traveler decides to stay one more night. When n_d is fixed, then differentiating (17) with respect to the round-trip time between the origin and the destination city, t_0, with the implicit function theorem yields the following proposition.

Proposition 1. *When the round-trip time increases by a small amount, then the marginal bias in benefit estimates of visiting a tourist city can be larger or smaller depending on the sign of the following formula,*

[7] The subscript "*c*" in *B* stands for "continuous", and indicates the benefits with a continuous trip length.

$$\frac{\partial \Delta}{\partial t_0} = \frac{\partial B_c}{\partial t_0} - \frac{\partial B_d}{\partial t_0}$$

$$= \underbrace{-\overbrace{wU_{t_i} \frac{\partial t_i(n_d^*)}{\partial t_0}}^{(-)}}_{[1]} \underbrace{-\overbrace{wU_k \frac{\partial k(n_d^*)}{\partial t_0}}^{(-)}}_{[2]} \underbrace{+\overbrace{p_i \frac{\partial k(n_d^*)}{\partial t_0}}^{(-)}}_{[3]} \underbrace{-\overbrace{(p_h + w\overline{T}_t)}^{(+)} \cdot \overbrace{\frac{\partial n_c^*}{\partial t_0}}^{(+)}}_{[4]}. \tag{18a}$$

Proof of proposition 1. See Appendix.

Proposition 1 states that when the round-trip travel time increases by a small amount, four effects described by terms [1]–[4] in (18a) cause the bias: terms [1] and [2] increase the bias, whereas terms [3] and [4] decrease the bias. Term [1] represents a decrease in utility through decreasing time spent on visiting recreation site *i* with an integer trip length. Term [2] indicates a decrease in utility through decreasing the number of visited recreation sites in the tourist city with an integer trip length. Term [3] is a decrease in costs for visiting recreation sites due to a decrease in the number of visiting sites with an integer trip length. Term [4] means an increase in the overnight expenditure and the opportunity cost due to an increase in continuous trip length.

Our model with an integer trip length exhibits the economic rationale behind the actual traveler's behavior. When the travel time increases by a small amount, the trip length will be fixed. The fixed trip length will induce the traveler to decrease time spent at each recreation site ($\partial t_i(n_d^*)/\partial t_0 < 0$) and the number of visited recreation sites ($\partial k(n_d^*)/\partial t_0 < 0$). When increasing an integer trip length, such as from two days to three days, t_i and k significantly increases. The significant increment makes the bias larger. This is why we have to take the integer property of trip length into account.

It is worth noting that with a continuous trip length, there is no change in time spent visiting recreation site *i* and the number of visited recreation sites ($\partial t_i^*/\partial t_0 = \partial k^*/\partial t_0 = 0$) even if the trip cost marginally increases, but these variables will decrease with an integer trip length. The reason is that the continuous trip length n_c can slightly increase according to the increase in the travel time t_0. Thus the traveler does not have to decrease t_i and k, and t_i and k remain unchanged. This mechanism is different from an actual traveler's behavior because when the travel time t_0 increases, the trip length does not increase unless the traveler decides to stay one more night.

3.2 Consumer surplus considering trip length as an integer

The travel cost method is often used to estimate economic benefits resulting from improving quality at a recreation site. The question is how this bias changes as the travel time t_0 increases. To answer this, we define the difference between benefit resulting from the improvement in quality of the destination city with a continuous trip length and that with an integer trip length:

$$\tilde{\Delta} \equiv \Delta B_c - \Delta B_d ,$$

where $\Delta B_i \equiv B_i(q^{before}) - B_i(q^{after}),\ \ i = \{c, d\}$, q^{before} and q^{after} are the quality before and after the improvement, respectively. Taking the partial derivative with respect to t_0 yields the following Corollary.

Corollary 1. *When the round-trip time increases by a small amount, then the marginal bias in estimates of benefits from improving the quality of a tourist city can be larger or smaller depending on the sign of the following formula,*

$$
\begin{aligned}
\frac{\partial\tilde{\Delta}}{\partial t_0} &= \frac{\partial \Delta B_c}{\partial t_0} - \frac{\partial \Delta B_d}{\partial t_0} \\
&= -wU_{t_i}\left(\overbrace{\frac{\partial t_j^{before}(n_d^*)}{\partial t_0}}^{(-)} - \overbrace{\frac{\partial t_i^{after}(n_d^*)}{\partial t_0}}^{(-)} \right) - wU_k \left(\overbrace{\frac{\partial k^{before}(n_d^*)}{\partial t_0}}^{(-)} - \overbrace{\frac{\partial k^{after}(n_d^*)}{\partial t_0}}^{(-)} \right) \qquad \text{(18b)} \\
&\quad + p_i \left(\overbrace{\frac{\partial k^{before}(n_d^*)}{\partial t_0}}^{(-)} - \overbrace{\frac{\partial k^{after}(n_d^*)}{\partial t_0}}^{(-)} \right) - \overbrace{(p_h + w\overline{T}_t)}^{(+)} \left(\overbrace{\frac{\partial n_c^{*before}}{\partial t_0}}^{(+)} - \overbrace{\frac{\partial n_c^{*after}}{\partial t_0}}^{(+)} \right).
\end{aligned}
$$

When estimating economic benefits resulting from improving quality in a tourist city, it is necessary to estimate the benefit based on the demand curve considering an integer trip length. In other words, instead of approximating the demand curve with a smooth line, it is necessary to consider how much the demand curve deviates from the smooth line according to (18b).

Next, we have to know how large the bias is when estimating benefits resulting from improving the quality of the recreation site. To answer this, the next section is devoted to quantifying $\tilde{\Delta}$ through numerical simulations.

4. Numerical Simulation

4.1 Specification of the utility function and parameter calibration

To quantify the benefits, we need to specify the functional form of the utility function. In real situations, travelers obtain different levels of utility at each recreation site. Based on this, we specify the utility function obtained from visiting recreation sites as

$$U(t_i,\ k;\ q) = \sum_{i=1}^{k} q \cdot \ln\left[\left(\frac{1}{\theta}\right)^i \cdot \left(t_i + 1\right)^{\alpha} \right], \qquad (19)$$

where θ is a parameter determining a choke price that drives the number of trips to zero, α is a parameter of t_i. Second, the utility obtained from consuming X is specified as

$$S(X) = \beta \ln(X), \qquad (20)$$

where β is a parameter.

Substituting (19) and (20) into (6) yields

$$\max_{\delta,t_i,k,n_d,X} w\delta q\left(\left(\frac{k(k+1)}{2}\right)\cdot\ln\left(\frac{1}{\theta}\right)+\alpha\ln\left(t_1+1\right)+\cdots+\alpha\ln\left(t_k+1\right)\right)$$
$$+w\beta\ln(X)+w\overline{T}-X-\delta\left(p_0t_0+\sum_{i=1}^{k}p_i+(n_d-1)p_h\right)-\delta wn_d\overline{T}_t \tag{21}$$

s.t. (2).

First we obtain the benefits with an integer trip length. Similar to (7), λ is the Lagrangian multiplier of (2); then, we obtain the first order conditions:

$$\frac{\partial L}{\partial t_i}=w\delta\left(\frac{\alpha}{t_i+1}q-\lambda\right)=0, \tag{22}$$

$$\frac{\partial L}{\partial k}=w\delta\left(\frac{2k+1}{2}q\cdot\ln\left(\frac{1}{\theta}\right)+q\alpha\ln\left(t_i+1\right)-\frac{p_i}{w}-\lambda\cdot t_i\right)=0, \tag{23}$$

$$\frac{\partial L}{\partial X}=w\beta\frac{1}{X}-1=0, \tag{24}$$

$$\frac{\partial L}{\partial \lambda}=-w\delta\left(t_0+\sum_{i=1}^{k}t_i-n_d\overline{T}_t\right)=0. \tag{25}$$

Substituting (19) and (20) with optimal variables obtained from the above first order conditions into (14) yields

$$B_d(q)=wq\left(\left(\frac{k(n_d^*)\cdot(k(n_d^*)+1)}{2}\right)\cdot\ln\left(\frac{1}{\theta}\right)+\alpha\sum_{i=1}^{k(n_d^*)}\ln\left(t_i(n_d^*)+1\right)\right)-p_0t_0-\sum_{i=1}^{k(n_d^*)}p_i-(n_d^*-1)p_h$$
$$-wn_d^*\overline{T}_t. \tag{26}$$

Next, we obtain the benefits with a continuous trip length. The first order condition with respect to n_c is identical to (15). Substituting the specified utility function with optimal variables into (16) yields

$$B_c(q)=wq\left(\left(\frac{k^*\cdot(k^*+1)}{2}\right)\cdot\ln\left(\frac{1}{\theta}\right)+\alpha\sum_{i=1}^{k^*}\ln\left(t_i^*+1\right)\right)-p_0t_0-\sum_{i=1}^{k^*}p_i-(n_c^*-1)p_h-wn_c^*\overline{T}_t.$$

(27)

4.2 Parameter calibration

The data we use in the simulation were obtained from a questionnaire survey on recreational activities in Hokkaido which was conducted in summer 2008.[8] We obtained detailed information of 131 visitors' personal characteristics and their behavior: t_0, n_d, k, and t_i.

The wage per hour is set as the time value used for domestic travelers in Japan (Ministry of Land, Infrastructure, Transport and Tourism; Civil Aviation Bureau, 2006): w = 3666 JPY.[9] The unit traveling expense between the origin and the destination is estimated as the relationship between the actual fare from each prefecture to Hokkaido and the actual traveling time: $p_0 = 4006.4t_0 + 6705.7$ in terms of JPY (t-values of the slope and the intercept are 2.73 and 0.57, respectively).

The model is abstract, but we attempt to represent the model as realistically as possible. We set some variables which are not available from the survey as follows: the daily available time excluding the time for sleeping and breakfast on a trip $\overline{T}_t$ = 10 hours; the daily overnight expenditure including the costs for breakfast and dinner p_h = 25,000 JPY; the cost of visiting recreation site i p_i = 1,000 JPY; the consumption of composite goods X = 9000 JPY. We suppose the situation where the quality of all recreation sites in Hokkaido improves by 3%, so let the initial quality be set as $q^{before} = 1$ and the quality after improvement as $q^{after} = 1.03$.

Next, we set parameters α and θ in utility function (19). According to the survey data, n_d is different among travelers. When taking the median of n_d from t_0 = 5.3 to 11.0 at one hour

[8] Hokkaido is one of the most popular sightseeing spots in Japan. In particular, visitors are attracted to Hokkaido for the coolness during the summer, and for skiing and other winter sports during the winter. Most travelers who visit Hokkaido go by airplane because Hokkaido is an island. The questionnaire survey was conducted at the New Chitose airport, which is the main gateway to Hokkaido, by members of the Yasuhisa Hayashiyama laboratory. The questionnaire survey data are shown in the Supplement.

[9] This value is obtained from the relationship between willingness to pay to save time and time saved (Preference approach method).

intervals, we can obtain the trend that n_d increases with t_0. The results show that the median of n_d is 2 when t_0 is less than 7.5 hours, the median of n_d is 3 when t_0 is between 7.5 and 8.5 hours, and the median of n_d is 4.5 when t_0 is between 8.5 and 9.5 hours, the median of n_d is 3 when t_0 is between 9.5 and 10.5 hours, and the median of n_d is 3 when t_0 is more than 10.5 hours. In addition, the medians of k and t_i are 4 and 9.1 respectively. We set the parameters α and θ to minimize the difference between the median of n_d, k, and t_i for the questionnaire survey data and our model's estimates n_d, k, and t_i. As a result, we calibrated those parameters as $\alpha = 16.18$ and $\theta = 7529.87$.

4.4 Results

We first show the estimated benefits with an integer trip length, summarized in Table 1. Table 1 shows results according to a change in travel time t_0 from 1 to 10. Each travel time has results of three cases: a one-day trip, a two-day trip, and a three-day trip. When $t_0 = 1$, if the household stays in the city for two days, then indirect utility would be the maximum; so, the optimal integer trip length is two days, and its benefit is 93,666 JPY. As t_0 gradually increases, the benefit of visiting the tourist city decreases and the optimal integer trip length changes to three days, at which $t_0 = 8$.

[Table 1 is here]

Next, supposing an improvement in quality of recreation sites in the tourist city, we show the magnitude of the bias resulting from ignoring the integer property of trip length, summarized in Table 2. It shows the integer and continuous trip length before and after the improvement in quality. ΔB_c is constant with the travel time, whereas ΔB_d varies according to the travel time. When $3 \le t_0 \le 7$, ΔB_c > ΔB_d and, as the rightmost column shows, ΔB_c is an 1–11% overestimation. When $t_0 \ge 8$, ΔB_c < ΔB_d, and, as the rightmost column shows, ΔB_c is a 8–14% underestimation. When $t_0 = 8$, the bias is 14 %, which is the maximum in this simulation. So, the benefit for those who live at $t_0 = 8$ is greatly biased.

[Table 2 is here.]

An intuitive reason for this result can be illustrated by Figure 1. The vertical axis is the benefits and the horizontal axis is the trip length. That is, inversed U-shape curves represent the benefits at each level of trip length. The benefit is maximized and the continuous trip length is optimal when the slope of the curve is zero. When improving quality of the destination city, the curve shifts obliquely right upward. This allows the benefit and the optimal continuous trip length to increase from B_c^{before} to B_c^{after}. The vertical difference between two maximum points at which the slope of the curve is zero indicates the benefit resulting from the improvement with a continuous trip length.

The benefit with an integer trip length is not identical to that with a continuous trip length. The mechanism of the underestimating results at $t_0 = 7$ is illustrated by Figure 1(a). When improving quality of the destination city, the integer trip length changes from 2 to 3. Intuitively, this represents a situation where the traveler really wants to stay for about 2.5 days with the improvement in the quality, but she cannot choose the continuous trip length and so she has to choose whether to stay for two days or three days. In this case, she feels that two days is too short, but three days is too long. Thus the increase in benefits is small compared to the case of continuous trip length.

The mechanism of underestimating results at $t_0 \geq 8$ is illustrated by Figure 1(c). The benefit with an integer trip length is equivalent to the vertical difference between two curves at the same integer trip length. Since n_d does not change even with the improvement in quality, the benefit with an integer trip length can be larger than that with a continuous trip length. Intuitively, this represents a situation where the traveler really wants to stay for about 2.5 days, but she cannot choose the continuous trip length and so she decides to choose three days. In other words, she has to stay an extra day, half of which is not so valuable to her. In this situation, she could use the extra day effectively if the quality of the tourist city were improved. That is why the increase in benefits with an *integer* trip length are greater than that with a *continuous* trip length.

This result implies that the benefit estimates could be biased in continuous trip length if the improvement in quality affects time spent at visited recreation sites in the tourist city.

[Figure 1 is here.]

4.5 Sensitivity analysis

The bias in subsection 4.4 is calculated based on the specific data on the trips to Hokkaido. To see in what kinds of cities benefit estimates would be biased, we explore sensitivity analyses in this subsection. Consider two cities: i) a city where the household takes a one-day trip to the city if the travel time is 3 hours or less by train, and the household takes a two-day trip if the travel time is 4 hours or more by train; ii) a city where the household takes a two-night trip (even if travel time is small). The question is how large the benefit bias is if the quality of the city increases by 3% for such places.

We change parameter α to 14.18 for city (i) and 18.18 for city (ii) to produce the above hypothetical sightseeing trips, setting the other parameters and exogenous variables as identical to the previous analysis (i.e., the case of Hokkaido in subsection 4.4 is baseline $\alpha = 16.18$).

The result is illustrated in Table 3. In city (i), when t_0 is 8 hours or more, the household does not go to the city because the benefit from the trip is negative. When $t_0 \leq 3$, ΔB_c is an 11–22% overestimation as the rightmost column shows. A mechanism of the overestimation is illustrated by Figure 1(b). In the continuous case, the trip length increases smoothly and continuously with the improvement in quality of the city. On the other hand, considering the integer trip length, it does not change even with the improvement. As a result, the increase in benefits is very small and the benefit with a continuous trip length is overestimated. When $t_0 \geq 4$, ΔB_c is a 8–21% underestimation whose mechanism is illustrated in Figure 1(a).

In city (ii), before the improvement, travelers stay three days regardless of the travel time t_0. After the improvement, she stays four days at only t_0 = 10. ΔB_c is an 2–9% underestimation when $t_0 \leq 4$, whose mechanism is shown in Figure 1(c). ΔB_c is an 2–9% overestimation when $6 \leq t_0 \leq 9$, whose mechanism is shown in Figure 1(b). ΔB_c is an 2% underestimation when t_0 = 10, whose mechanism is shown in Figure 1(d).

Figure 2 shows the relationship between α and the maximum bias with each α. This figure indicates that the bias becomes smaller with α, that is, the difference in travel behavior between the continuous case and the integer case is reduced. Conversely, the gap between one-day trip and two-day trip is large, which can make the benefit estimates biased bymore than 20%. Since these biases based on the real travel behavior are not negligible, this analysis can conclude that it is necessary for TCM to consider the integer characteristics of trip length.

[Table 3 is here.]

5. Conclusion

This paper has explored the effect of the integer property in the travel cost method in terms of the number of days spent in a tourist city. We have shown that ignoring the integer property of trip length theoretically and numerically generates a significant bias in the benefit estimates.

We have identified the reason why the bias could arise. When travel time increases by a small amount, the integer trip length cannot change unless the household decides to stay one more night. The fixed integer trip length will induce the traveler to decrease time spent on visiting recreation sites. However, the continuous trip length can slightly increase without decreasing time spent visiting recreation sites. Such differences cause the bias in benefit estimates. That is, as Dobbs (1993) pointed out for the case of the integer property of the number of trips, the benefit estimated by the integration of a smooth demand function has a bias for the case of the integer property of trip length in terms of days.

When measuring benefits resulting from an improvement in quality at visited recreation sites, our numerical simulation shows that the benefits with a continuous trip length is as much as a 22% overestimation. So, when TCM is applied to trips with overnight stays, we have to explicitly treat trip length as an integer variable.

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Appendix. Proof of proposition 1

First, we investigate the sign of $\frac{\partial t_i(n_d^*)}{\partial t_0}$ and $\frac{\partial k(n_d^*)}{\partial t_0}$. Total differentiating (8), (9) and (11) with respect to endogenous variables except n_d and travel time t_0, we can obtain

$$\underbrace{\begin{pmatrix} \frac{\partial^2 U_S}{\partial t_i^2} & 0 & -1 \\ -\lambda & \frac{\partial^2 U_S}{\partial k^2} & -t_i \\ -1 & -t_i & 0 \end{pmatrix}}_{\equiv A_d} \begin{pmatrix} dt_i \\ dk \\ d\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} dt_0 .$$

Determinant of A_d is positive because we assume the second order conditions for optimization are satisfied:

$$|A_d| = -\frac{\partial^2 U_S}{\partial t_i^2} t_i^2 - \lambda t_i - \frac{\partial^2 U_S}{\partial k^2} > 0 .$$

Using Cramer's rule,

$$\frac{\partial t_i(n_d^*)}{\partial t_0} = \underbrace{\frac{1}{|A_d|}}_{(+)}\left(\underbrace{\frac{\partial^2 U_S}{\partial k^2}}_{(-)}\right) < 0\,.$$

From the second order condition,

$$\left(\frac{\partial^2 U_S}{\partial t_i^{\,2}} t_i + \lambda\right) < \underbrace{\frac{\partial^2 U_S}{\partial k^2}\Big/ t_i}_{(-)}\,.$$

Using Cramer's rule with the above inequality,

$$\frac{\partial k(n_d^*)}{\partial t_0} = \underbrace{\frac{1}{|A_d|}}_{(+)}\underbrace{\left(\frac{\partial^2 U_S}{\partial t_i^{\,2}} t_i + \lambda\right)}_{(-)} < 0.$$

Next, we investigate the sign of $\frac{\partial t_i^*}{\partial t_0}$, $\frac{\partial k^*}{\partial t_0}$, and $\frac{\partial n_c^*}{\partial t_0}$. Total differentiating (8), (9), (11), and (15), we can obtain

$$\underbrace{\begin{pmatrix} \frac{\partial^2 U_S}{\partial t_i^{\,2}} & 0 & 0 & -1 \\ -\lambda & \frac{\partial^2 U_S}{\partial k^2} & 0 & -t_i \\ 0 & 0 & 0 & \overline{T}_t \\ -1 & -t_i & \overline{T}_t & 0 \end{pmatrix}}_{\equiv A_C} \begin{pmatrix} dt_i \\ dk \\ dn_c \\ d\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} dt_0.$$

Determinant of A_c is negative because we assume the second order conditions for optimization are satisfied. Using Cramer's rule, $\frac{\partial t_i^*}{\partial t_0} = 0$, $\frac{\partial k^*}{\partial t_0} = 0$, and

$$\frac{\partial n_c^*}{\partial t_0} = \underbrace{\frac{1}{|A_c|}}_{(-)}\left[-\underbrace{\frac{\partial^2 U_S}{\partial t_i^{\,2}}}_{(-)}\underbrace{\frac{\partial^2 U_S}{\partial k^2}}_{(-)}\overline{T}_t\right] > 0\,. \,\|$$

Table 1. The recreational benefits with an integer trip length.

Travel time t_0 (hour)	Integer trip length n_d	Time spent at each site t_i (hour)	Number of sites visited k	Indirect utility	Benefits with an integer trip length B_d
1	1	6.1	1.5	196,376	-
	2	9.3	2.1	203,271	93,666
	3	11.9	2.4	192,928	-
2	1	5.8	1.4	183,823	-
	2	9.0	2.0	193,398	83,793
	3	11.7	2.4	184,283	-
⋮	⋮	⋮	⋮	⋮	⋮
7	1	7.5	1.7	111,631	-
	2	10.4	2.2	141,153	31,548
	3	12.9	2.6	139,539	-
8	2	7.1	1.7	130,001	-
	3	10.1	2.2	130,247	20,642
	4	12.7	2.5	116,621	-
⋮	⋮	⋮	⋮	⋮	⋮
9	2	6.8	1.6	118,557	-
	3	9.8	2.1	120,823	11,218
	4	12.4	2.5	108,231	-
10	2	6.5	1.5	106,788	-
	3	9.5	2.1	111,259	1,655
	4	12.2	2.5	99,761	-

Table 2. The bias in benefit estimates due to ignoring the integer property of trip length
$\alpha = 16.18$ (baseline)

t_0	n_d		ΔB_d	n_c		ΔB_c	$\Delta B_d / \Delta B_c$
	Before	After		Before	After		
1	2	2	6143	1.78	1.88	5837	1.05
2	2	2	5965	1.88	1.98	5837	1.02
3	2	2	5783	1.98	2.08	5837	0.99
4	2	2	5594	2.08	2.18	5837	0.96
5	2	2	5400	2.18	2.28	5837	0.93
6	2	2	5199	2.28	2.38	5837	0.89
7	2	3	5193	2.38	2.48	5837	0.89
8	3	3	6647	2.48	2.58	5837	1.14
9	3	3	6483	2.58	2.68	5837	1.11
10	3	3	6315	2.68	2.78	5837	1.08

Table 3. The bias in benefit estimates due to ignoring the integer property of trip length.
(i) $\alpha = 14.18$ (city (i) in sensitivity analysis)

t_0	n_d		ΔB_d	n_c		ΔB_c	$\Delta B_d / \Delta B_c$
	Before	After		Before	After		
1	1	1	3446	1.18	1.25	3865	0.89
2	1	1	3237	1.28	1.35	3865	0.84
3	1	1	3016	1.38	1.45	3865	0.78
4	2	2	4676	1.48	1.55	3865	1.21
5	2	2	4519	1.58	1.65	3865	1.17
6	2	2	4358	1.68	1.75	3865	1.13
7	2	2	4190	1.78	1.85	3865	1.08

(ii) $\alpha = 18.18$ (city (ii) in sensitivity analysis)

t_0	n_d		ΔB_d	n_c		ΔB_c	$\Delta B_d / \Delta B_c$
	Before	After		Before	After		
1	3	3	9108	2.52	2.66	8387	1.09
2	3	3	8938	2.62	2.76	8387	1.07
3	3	3	8764	2.72	2.86	8387	1.04
4	3	3	8587	2.82	2.96	8387	1.02
5	3	3	8406	2.92	3.06	8387	1.00
6	3	3	8221	3.02	3.16	8387	0.98
7	3	3	8031	3.12	3.26	8387	0.96
8	3	3	7838	3.22	3.36	8387	0.93
9	3	3	7639	3.32	3.46	8387	0.91
10	3	4	8530	3.42	3.56	8387	1.02

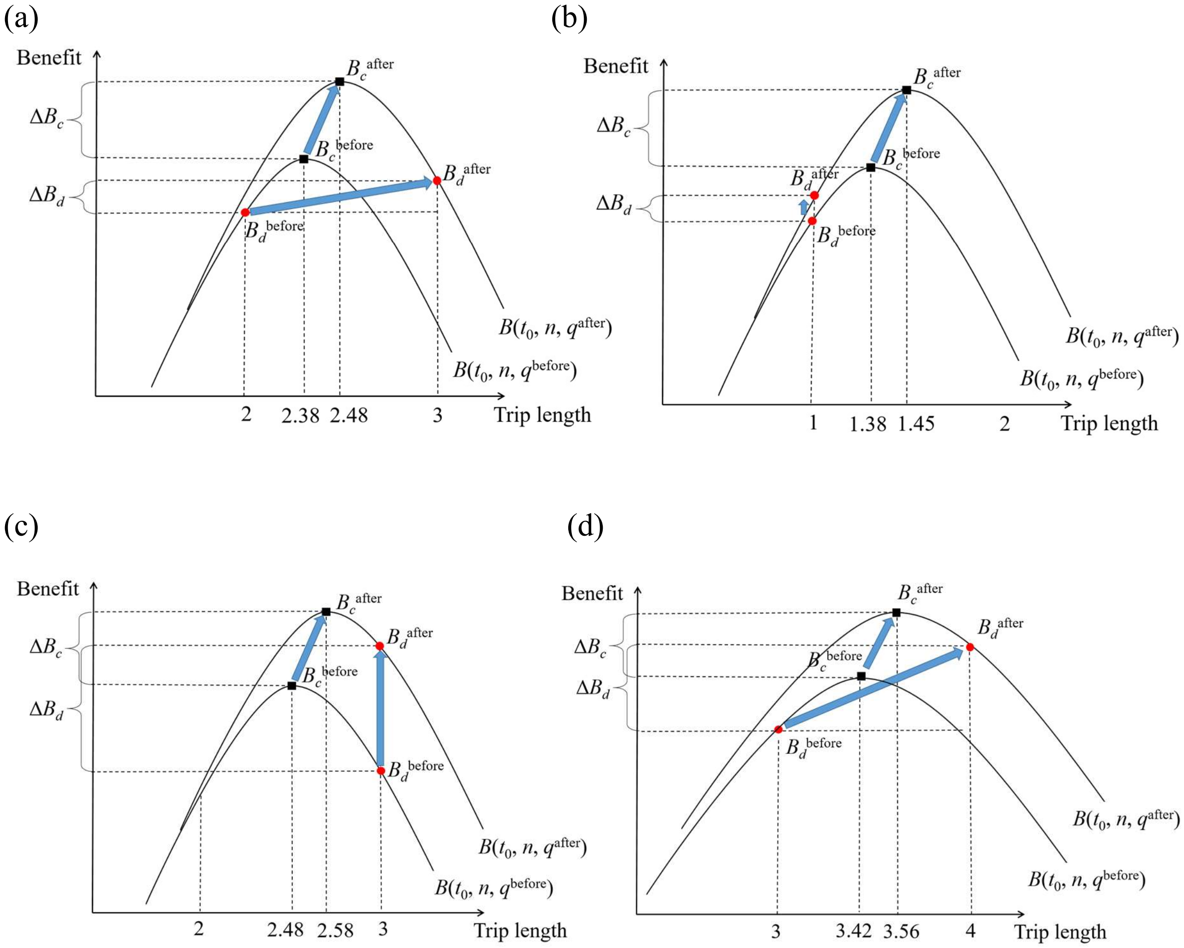

Figure 1. Mechanism of the bias in benefit estimates resulting from changes in the quality at recreation sites; the upper figures (a) and (b) show overestimated results; the lower figures (c) and (d) show underestimated results.

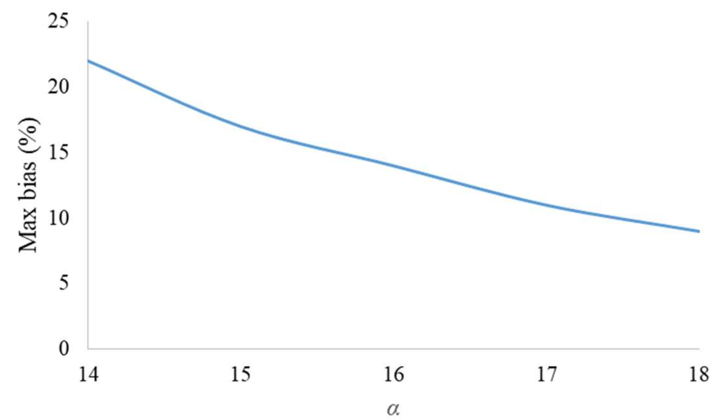

Figure 2. Relationship between α and maximum bias in benefit estimates with each α. Note that the max. bias does not distinguish between overestimates and underestimates.

Supplement. Questionnaire survey data

Origin	Number of samples	Travel time (hour)	Travel fee (yen)	Median trip length#1 (days)	Median sites visited#1	Average time on site#2 (hour)
Miyagi	2	5.3	30,420	3	7	5.5
Akita	2	6.7	27,800	2	3	9.1
Fukushima	1	7.1	33,750	3	4	9.2
Ibaraki	1	10.9	25,330	4	3	14.4
Gunma	1	11.1	24,470	3	4	8.2
Saitama	8	6.8	21,980	3	5	7.4
Chiba	7	7.3	22,340	2	3	8.9
Tokyo	17	5.9	21,560	3	3	12.7
Kanagawa	14	6.0	21,560	2	4	7.0
Niigata	1	5.3	31,100	2	4	7.2
Toyama	2	6.6	35,700	2	5	5.5
Ishikawa	1	9.1	37,280	5	7	7.8
Fukui	1	8.0	36,520	3	4	9.0
Gifu	4	7.2	19,110	2	4	6.7
Shizuoka	2	8.1	27,550	3	5	7.2
Aichi	22	6.1	18,820	2	3	9.3
Mie	2	6.1	44,070	2	2	14.0
Kyoto	4	7.7	44,440	3	2	18.2
Osaka	7	7.0	44,310	2	4	6.8
Hyogo	12	6.4	20,120	3	3	12.5
Nara	4	8.2	44,940	3	5	7.2
Wakayama	1	7.0	44,340	3	4	9.3
Tottori	1	9.2	53,590	5	10	5.5
Shimane	1	8.5	55,840	5	3	18.5
Okayama	1	9.3	26,100	2	4	6.2
Hiroshima	6	9.0	49,410	4	4	11.2
Yamaguchi	1	9.7	60,230	5	5	10.9
Ehime	1	8.5	63,100	2	3	8.5
Kochi	1	10.2	55,630	2	4	6.0
Fukuoka	1	6.5	53,550	3	5	7.5
Kumamoto	1	10.2	53,000	3	7	4.8
Kagoshima	1	10.6	73,700	3	6	5.6

#1: This "median" is not strictly median, but a rough definition of median in accordance with our purpose. To represent the characteristics of the trips from each origin, we determine the number of trips as follows, using the sample data. For example, two samples have 3 and 4. In this situation, we chose 4 as the median because we need an integer number. When we have 6 and 8, we chose 7 as the median. This rough definition was necessary because the number of samples is small. But this definition likely does not affect our result to a large extent.

#2: Time on site is calculated by dividing the available time by the number of median sites visited. The available time is the number of days multiplied by 10 hours (=24hours-14hours).